An algebraic framework for the design of nonlinear observers with unknown inputs

Jean-Pierre Barbot, Michel Fliess, Thierry Floquet

An algebraic framework for the design of nonlinear observers with unknown inputs

Jean-Pierre BARBOT, Michel FLIESS and Thierry FLOQUET

Abstract— **The observability properties of nonlinear systems with unknown inputs are characterized via differentially algebraic techniques. State variables and unknown inputs are estimated thanks to a new algebraic numerical differentiator. It is shown through an academic example and a concrete case-study that the proposed scheme can be applied to systems that fail to fulfill some usual structural assumptions.**

I. INTRODUCTION

In this note, the problem of designing unknown input observers for nonlinear MIMO systems is discussed from a differentially algebraic standpoint. The design of an observer for a multivariable system partially driven by unknown inputs has been widely studied in the literature. Indeed, specific applications of asymptotic and finite time observers for systems with unknown inputs can be found in a large number of fields such as: robust reconstruction of state variables for systems subject to exogenous disturbances, parameter identification, fault detection and identification or secure communication.

The design of observers for nonlinear systems is a challenging problem (even for accurately known systems) that has received a considerable amount of attention. In most approaches, nonlinear coordinate transformations are employed to transform the nonlinear system into block triangular observer canonical forms[1]. Then, high gain [17], backstepping [28], or sliding mode observers [2], [31], [42] can be designed. Some of them are concerned with applications in the field of fault detection and identification, in particular the nonlinear Fundamental Problem of Residual Generation (FPRG) [15], [18], [32]. Actually, structural conditions have to be met such that the nonlinearities and the unknown inputs act only on the last dynamics of each triangular form. This *unknown input-output injection quasi-linearization* problem is similar to the feedback linearization problem for MIMO systems (see [23], Chapter 5). In particular, this implies that some coupling matrix between the outputs and the unknown inputs should be invertible. This assumption is also known as the observability matching condition. One can refer to [16] for a formulation of this condition in terms of observability indices and the unknown input characteristic indexes (or relative degrees).

In this paper, a differentially algebraic approach, which is extending a module-theoretic setting for linear systems in [5], is considered for the design of unknown input observers. A necessary and sufficient condition for their existence is given in terms of input-output invertibility and zero dynamics. Note that the design of unknown input observers is rarely seen as a left invertibility problem[2]. In general, one usually tries to compute left inverse systems in input reconstruction problems [21] (where observing the whole state variables is not necessarily required) and algorithms that provide such inverse systems are given for instance in [6], [7], [39]. The approach given in this paper specifically shows that the restrictive structure of observers, imposed by the matching condition, can be weakened if additional independent output signals are generated by means of differentiators of the available measurements. For this, a new type of non-asymptotic observers, also based on differential algebra concepts and already developed in [13], is recalled. The given procedure is illustrated via an academic example and an application to secure communication.

II. AN ALGEBRAIC SETTING FOR NONLINEAR OBSERVABILITY[3]

A. Differential algebra

Commutative algebra, which is mainly concerned with the study of commutative rings and fields, provides the right tools for understanding algebraic equations (see, e.g., [10]). *Differential algebra*, which was mainly founded by Ritt [35] and Kolchin [25], extends to differential equations concepts and results from commutative algebra[4].

1) Basic Definitions: A *differential ring* R (see, e.g., [25] and [4]) will be here a commutative ring[5] of characteristic

The authors are supported by the *Projet ALIEN, INRIA-Futurs*.

J.P. Barbot is with Équipe Commande des Systèmes (ECS), ENSEA, 6 Avenue du Ponceau, 95014 Cergy, France. barbot@ensea.fr

M. FLiess is with Équipe MAX, LIX (CNRS, UMR 7161), École polytechnique, 91128 Palaiseau, France. Michel.Fliess@polytechnique.edu

T. Floquet is with LAGIS (CNRS, UMR 8146), École Centrale de Lille, BP 48, Cité Scientifique, 59651 Villeneuve-d'Ascq, France Thierry.Floquet@ec-lille.fr

[1] In the case of linear systems, easily verifiable system theoretic conditions, which are necessary and sufficient for the existence of a linear unknown input observer, have been established (see e.g. [22] and the reference therein). One possible statement of these conditions is that the transfer function matrix between the unmeasurable input and the measured outputs must be minimum phase and of relative degree one.

[2] See [34] for a survey about left invertibility of nonlinear control systems, see [3], [37], [38], [45] for pioneering works on the existence the construction of an inverse system in the linear case, and see [24], [39] for generalizations to nonlinear systems. We follow here the differentially algebraic setting [11] where necessary and sufficient conditions were expressed in terms of the *differential output rank.*

[3] See also [13], [14], [36], [40] for more details on the differential algebraic approach to nonlinear systems.

[4] Algebraic equations are differential equations of order 0.

[5] See, e.g., [1], [4] for basic notions in commutative algebra.

zero, which is equipped with a single *derivation* $\frac{d}{dt} : R \to R$ such that, for any $a, b \in R$,

- $\frac{d}{dt}(a+b) = \dot{a} + \dot{b}$,
- $\frac{d}{dt}(ab) = \dot{a}b + a\dot{b}$.

where $\frac{da}{dt} = \dot{a}$, $\frac{d^\nu a}{dt^\nu} = a^{(\nu)}$, $\nu \geq 0$. A *differential field* is a differential ring which is a field. A *constant* of R is an element $c \in R$ such that $\dot{c} = 0$. A *(differential) ring* (resp. *field*) *of constants* is a differential ring (resp. field) which only contains constants. The set of all constant elements of R is a subring (resp. subfield), which is called the *subring* (resp.*subfield*) *of constants*.

2) Field extensions: All fields are assumed to be of characteristic zero. A differential field extension K/k is given by two differential fields k, K, such that $k \subseteq K$. Write $k\langle S\rangle$ the differential subfield of K generated by k and a subset $S \subset K$. Assume also that the differential field extension K/k is *finitely generated*, i.e., there exists a finite subset $S \subset K$ such that $K = k\langle S\rangle$. An element a of K is said to be *differentially algebraic* over k if, and only if, it satisfies an algebraic differential equation with coefficients in k: there exists a non-zero polynomial P over k, in several indeterminates, such that $P(a, \dot{a}, \ldots, a^{(\nu)}) = 0$. It is said to be *differentially transcendental* over k if, and only if, it is not differentially algebraic. The extension K/k is said to be *differentially algebraic* if, and only if, any element of K is differentially algebraic over k. An extension which is not differentially algebraic is said to be *differentially transcendental*.

The following result is playing an important rôle:

Proposition 2.1: The extension K/k is differentially algebraic if, and only if, its transcendence degree is finite.

A set $\{\xi_\iota \mid \iota \in I\}$ of elements in K is said to be *differentially algebraically independent* over k if, and only if, the set $\{\xi_\iota^{(\nu)} \mid \iota \in I, \nu \geq 0\}$ of derivatives of any order is algebraically independent over k. If a set is not differentially algebraically independent over k, it is *differentially algebraically dependent over* k. An independent set which is maximal with respect to inclusion is called a *differential transcendence basis*. The cardinalities, i.e., the numbers of elements, of two such bases are equal. This cardinality is the *differential transcendence degree* of the extension K/k; it is written diff tr deg (K/k). Note that this degree is 0 if, and only if, K/k is differentially algebraic.

B. Nonlinear systems

1) Generalities: Let k be a given differential ground field. A *(nonlinear) (input-output) system* is a finitely generated differential extension K/k, where K contains the sets $\boldsymbol{u} = (u_1, \ldots, u_m)$ and $\boldsymbol{y} = (y_1, \ldots, y_p)$ of control and output variables, and such that the extension $K/k\langle\boldsymbol{u}\rangle$ is differentially algebraic. Assume for simplicity's sake that the control variables are *independent*, i.e., that $\boldsymbol{u}$ is a differential transcendence basis of the extension $k\langle\boldsymbol{u}\rangle/k$.

Remark 2.1: Remember that differential algebra considers algebraic differential equations, i.e., differential equations which only contain polynomial functions of the variables and their derivatives up to some finite order. This is of course not always the case in practice. In the example of Section IV, for instance, appears the transcendental function $\sin\theta$. As already noted in [14], we recover algebraic differential equations by introducing $\tan\frac{\theta}{2}$.

2) State-variable representation: We know, from proposition 2.1, that the transcendence degree of the extension $K/k\langle\boldsymbol{u}\rangle$ is finite, say n. Let $\boldsymbol{x} = (x_1, \ldots, x_n)$ be a transcendence basis. Any derivative $\dot{x}_i$, $i = 1, \ldots, n$, and any output variable y_j, $j = 1, \ldots, p$, are algebraically dependent over $k\langle\boldsymbol{u}\rangle$ on $\boldsymbol{x}$:

$$\begin{array}{ll} A_i(\dot{x}_i, \boldsymbol{x}) = 0 & i = 1, \ldots, n \\ B_j(y_j, \boldsymbol{x}) = 0 & j = 1, \ldots, p \end{array} \tag{1}$$

where $A_i \in k\langle\boldsymbol{u}\rangle[\dot{x}_i, \boldsymbol{x}]$, $B_j \in k\langle\boldsymbol{u}\rangle[y_j, \boldsymbol{x}]$, i.e., the coefficients of the polynomials A_i, B_j depend on the control variables and their derivatives up to some finite order.

Remark 2.2: Note the difference with the usual state-variable representation

$$\begin{array}{l} \dot{\boldsymbol{x}} = F(\boldsymbol{x}, \boldsymbol{u}) \\ \boldsymbol{y} = H(\boldsymbol{x}) \end{array}$$

3) Invertibility and zero dynamics: System K/k is said to be *left* (resp. *right*) *invertible* [11] if, and only if, the differential transcendence degree of the extension $k\langle y\rangle/k$ is equal to m (resp. p). It is easy to check that left invertibility is equivalent to the fact that the extension $K/k\langle y\rangle$ is differentially algebraic. If the system is *square*, i.e., $m = p$, left and right invertibilities coincide and the system is said to be *invertible*. Call the extension $K/k\langle y\rangle$ the *zero*, or the *inverse*, *dynamics* (compare with [23]). This dynamics is said to be *trivial* if, and only if, the extension $K/k\langle y\rangle$ is algebraic. The next result is clear:

Proposition 2.2: An input-output system with a trivial zero dynamics is left invertible.

4) Differential flatness: System K/k is said to be *(differentially) flat* (see, e.g., [14], [36], [40]) if, and only if, the algebraic closure $\bar{K}$ of K is equal to the algebraic closure of a purely differentially transcendental extension of k. It means in other words that there exists a finite subset $\boldsymbol{z} = \{z_1, \ldots, z_m\}$ of $\bar{K}^{\text{ pure}}$ such that

- $z_1, \ldots, z_m$ are differentially algebraically independent over k,
- $z_1, \ldots, z_m$ are algebraic over K,
- any system variable is algebraic over $k\langle z_1, \ldots, z_m\rangle$.

$\boldsymbol{z}$ is a *flat*, or *linearizing*, *output*. For a flat dynamics, it is known that the number m of its elements is equal to the number of independent control variables. Assume that K/k is an input-output system. The next property is clear:

Proposition 2.3: An input-output system is flat and its output is a flat output if, and only if, it is square with a trivial zero dynamics.

C. Observability and reconstructors

1) Generalities: According to [8], [9], system K/k is said to be *observable* if, and only if, the extension $K/k\langle\boldsymbol{u}, \boldsymbol{y}\rangle$ is algebraic. It means in plain words that observability is equivalent to the following fact: Any system variable, a state variable for instance, may be expressed as a function of the input and output variables and of their derivatives up to some finite order. Call this function a *reconstructor* of the corresponding system variable. This new definition of observability is "roughly" equivalent (see [8], [9] for details[6]) to its usual differential geometric counterpart due to [19].

2) Unknown inputs: A system variable $\xi \in K$ is said to be *observable with unknown inputs* if, and only if, it belongs to the algebraic closure of $k\langle \boldsymbol{y} \rangle$. More generally, the system K/k is said to be *observable with unknown inputs* if, and only if, the algebraic closure of $k\langle \boldsymbol{y} \rangle$ in K coincides with K. It means in other words that any system variable, a state variable or an input variable for instance, can be expressed as function of the output variables and their derivatives up to some finite order. Call this function the *input-free reconstructor* or the *fast input-free observer* of the reconstructor. The following property follows at once from Propositions 2.2 and 2.3:

Proposition 2.4: An input-output system is observable with unknown inputs if, and only if, its zero dynamics is trivial. If moreover the system is square, it is flat and its output is a flat output.

III. Numerical differentiation[7]

A. Polynomial time signals

Consider the real-valued polynomial function $x_N(t) = \sum_{\nu=0}^{N} x^{(\nu)}(0)\frac{t^\nu}{\nu!} \in \mathbb{R}[t]$, $t \geq 0$, of degree N. Rewrite it in the well known notations of operational calculus (see, e.g., [44]):

$$X_N(s) = \sum_{\nu=0}^{N} \frac{x^{(\nu)}(0)}{s^{\nu+1}}$$

We know utilize $\frac{d}{ds}$, which corresponds in the time domain to the multiplication by $-t$. Multiply both sides by $\frac{d^\alpha}{ds^\alpha} s^{N+1}$, $\alpha = 0, 1, \ldots, N$. The quantities $x^{(\nu)}(0)$, $\nu = 0, 1, \ldots, N$ are given by the triangular system of linear equations:

$$\frac{d^\alpha s^{N+1} X_N}{ds^\alpha} = \frac{d^\alpha}{ds^\alpha}\left(\sum_{\nu=0}^{N} x^{(\nu)}(0)s^{N-\nu}\right) \tag{2}$$

The time derivatives, i.e., $s^\mu \frac{d^\iota X_N}{ds^\iota}$, $\mu = 1, \ldots, N$, $0 \leq \iota \leq N$, are removed by multiplying both sides of Equation (2) by $s^{-\bar{N}}$, $\bar{N} > N$.

B. Analytic time signals

Consider a real-valued analytic time function defined by the convergent power series $x(t) = \sum_{\nu=0}^{\infty} x^{(\nu)}(0)\frac{t^\nu}{\nu!}$, where $0 \leq t < \rho$. Approximate $x(t)$ in the interval $(0, \varepsilon)$, $0 < \varepsilon \leq \rho$, by its truncated Taylor expansion $x_N(t) = \sum_{\nu=0}^{N} x^{(\nu)}(0)\frac{t^\nu}{\nu!}$ of order N. Introduce the operational analogue of $x(t)$, i.e., $X(s) = \sum_{\nu \geq 0} \frac{x^{(\nu)}(0)}{s^{\nu+1}}$. Denote by $[x^{(\nu)}(0)]_{e_N}(t)$, $0 \leq \nu \leq N$, the numerical estimate of $x^{(\nu)}(0)$, which is obtained by replacing $X_N(s)$ by $X(s)$ in Eq. (2). It can be shown [13] that a good estimate is obtained in this way.

C. Noisy signals

Assume that our signals are corrupted by additive noises. Those noises are viewed here as highly fluctuating, or oscillatory, phenomena. They may be therefore attenuated by low-pass filters, like iterated time integrals. Remember that those iterated time integrals do occur in Equation (2) after multiplying both sides by $s^{-\bar{N}}$, for $\bar{N} > 0$ large enough.

Remark 3.1: See [12] for a precise mathematical foundation, which is based on *nonstandard analysis*. A highly fluctuating function of zero mean is then defined by the following property: its integral over a finite time interval is *infinitesimal*, i.e., "very small"[8].

Remark 3.2: Resetting and utilizing sliding time windows permit to estimate derivatives of various orders at any sampled time instant.

IV. Examples

In this section, two examples, that do not satisfy the conditions required by usual nonlinear unknown input observers, are presented.

A. A flat system

The following nonlinear system

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{pmatrix} = \begin{pmatrix} x_2 \\ -\sin(x_2) - x_4 + m_1 \\ x_4 + m_1 \\ -x_3 + \cos(x_3) + m_2 \end{pmatrix} \tag{3}$$

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = \begin{pmatrix} x_1 \\ x_3 \end{pmatrix} \tag{4}$$

where m_1, m_2 are unknown input variables, has relative degree $\rho = \{2, 1\}$ with respect to the output $y = [y_1, y_2]^T$ and the coupling matrix

$$\Gamma(x) = \begin{pmatrix} L_{g_1} L_f^{\rho_1 - 1} h_1(x) & L_{g_2} L_f^{\rho_1 - 1} h_1(x) \\ L_{g_1} L_f^{\rho_2 - 1} h_2(x) & L_{g_2} L_f^{\rho_2 - 1} h_2(x) \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}$$

is singular. According to [16], (3)-(4) cannot be transformed into a canonical block triangular observable form. Usual observers fail therefore to estimate the state.

Easy but lengthy computations, which will not be reported here due to a lack of space, yield

$$\begin{aligned} x_2 &= \dot{y}_1 \\ x_4 &= \frac{1}{2}\left(\dot{y}_2 - \ddot{y}_1 - \sin(\dot{y}_1)\right) \\ m_1 &= \frac{1}{2}\left(\dot{y}_2 + \ddot{y}_1 + \sin(\dot{y}_1)\right) \\ m_2 &= \frac{1}{2}\left(\ddot{y}_2 - y_1^{(3)} - \ddot{y}_1 \cos(\dot{y}_1)\right) + y_2 - \cos(y_2) \end{aligned}$$

They show according to Section II-B.4 that the system (3)-(4) is flat and that the couple $\{y_1, y_2\}$ is a flat output.

[6] The differential algebraic and the differential geometric languages are not equivalent. We cannot therefore hope for a "one-to-one bijection" between definitions and results which are expressed in those two settings.

[7] See [13], [29] for more details and for related references on numerical differentiation.

[8] This approach applies as well to multiplicative noises (see [12]). The assumption on the noises being only additive is therefore unnecessary.

Moreover the above equations may be interpreted according to Section II-C.2 as input-free reconstructors or fast input-free observers.

The simulation results given Fig. 1, 2, 3 and 4 were obtained using fast algebraic differentiators. Random noises were added on both outputs.

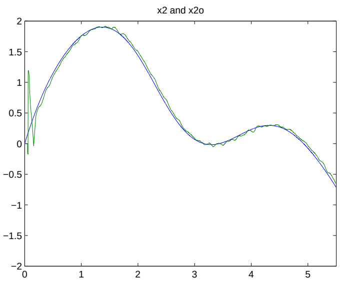

Fig. 1. x_2 and its estimate

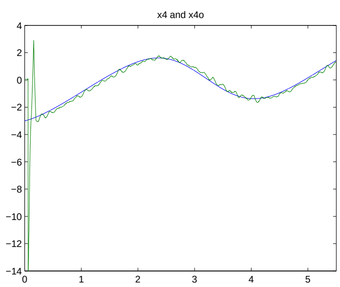

Fig. 2. x_4 and its estimate

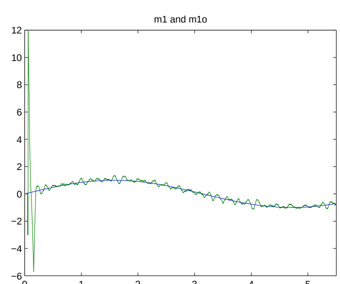

Fig. 3. m_1 and its estimate

B. Application to secure communication

Since the work of Pecora and Caroll [30], it is known that the problem of secure communication can be investigated with respect to the synchronization of chaotic systems (for usual cryptography methods see, e.g., [20]). The idea is to use the output of a particular dynamical chaotic system (that masks the message) to drive the response of a generally identical system (that recovers the message) so that they oscillate in a synchronized manner. Secure communication by the synchronization of chaotic systems has different design methods such as: addition (see, e.g., [27]), chaotic

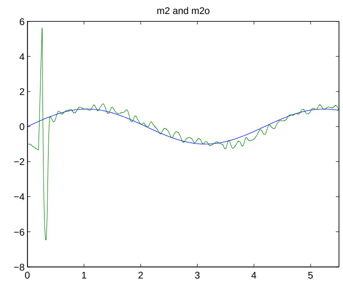

Fig. 4. m_2 and its estimate

switch keying (see, e.g., [26]), chaotic modulation (see, e.g., [43]),...

Hereafter is highlighted the interest of our approach for the chaotic modulation method in the case of multi-input multi-output system. Note that the algebraic viewpoint (as well as the use of the algebraic derivative method for the efficient and fast computation of accurate approximations to the successive time derivatives of the transmitted observable output) has already been successfully developed for the state estimation problem associated with the chaotic encryption-decoding problem in the case of some classes of nonlinear systems [41].

Let us consider the following chaotic system given in [33]:

$$\begin{aligned}
\dot{x}_1 &= a(x_2 - x_1) + x_2 x_3 \\
\dot{x}_2 &= b(x_1 + x_2) - x_1 x_3 \\
\dot{x}_3 &= -c x_3 - e x_4 + x_1 x_2 \\
\dot{x}_4 &= -d x_4 + f x_3 + x_1 x_3
\end{aligned}$$

In order to send the confidential messages m_1 and m_2, the following transmitter is designed:

$$\begin{aligned}
\dot{x}_1 &= a(x_2 - x_1) + x_2 x_3 + \mu_1(x_1, x_2, x_3) m_1 \\
\dot{x}_2 &= b(x_1 + x_2) - x_1 x_3 \\
\dot{x}_3 &= -c x_3 - e x_4 + x_1 x_2 + \mu_2(x_1, x_2, x_3) m_1 \\
\dot{x}_4 &= -d x_4 + f x_3 + x_1 x_3 + \mu_3(x) m_1 + \mu_4(x) m_2
\end{aligned}$$

The chosen outputs are $y_1 = x_1$, $y_2 = x_2$ and m_1 and m_2 should be seen as two unknown inputs. This system fails to fulfil the observability matching condition. Thus, the complexity of the encryption algorithm is increased, because well known observer-based approach can not be used here. In this example, the key (similarly to a usual ciphering problem) is the parameter vector $\begin{bmatrix} a & b & c & d & e & f \end{bmatrix}^T$, and the μ_i are chosen in order to guarantee the confidentiality and such that the chaotic behavior of the system is not destroyed. For the confidentiality of the system, it is important for example to verify that the parameters are not identifiable with known inputs (known-plaintext attack).

As in Section IV-A, easy calculations yield

$$
\begin{aligned}
x_3 &= \frac{b(y_1+y_2)-\dot{y}_2}{y_1} \\
m_1 &= \frac{1}{\tilde{\mu}_1(y_1,y_2,\dot{y}_2)}\left(\dot{y}_1 - a(y_1-y_2) + \frac{y_2 y_a}{y_1}\right) \\
x_4 &= \frac{1}{e}(-c\frac{y_a}{y_1} + y_1 y_2 + \tilde{\mu}_2(y_1,y_2,\dot{y}_2)m_1) \\
m_2 &= \frac{\dot{x}_4 + dx_4 - f\frac{y_a}{y_1} + y_a - \tilde{\mu}_3(y_1,y_2,\dot{y}_1,\dot{y}_2,\ddot{y}_2)m_1}{\tilde{\mu}_2(y_1,y_2,\dot{y}_1,\dot{y}_2,\ddot{y}_2)}
\end{aligned}
\tag{5}
$$

where $y_a = x_1 x_3 = b(y_1+y_2) - \dot{y}_2$ and $\tilde{\mu}_i(.) = \mu_i(.)$.
The simulation results were obtained with the following parameters $a = 42.5$, $b = 24$, $c = 13$, $d = 20$, $e = 50$, $f = 40$, $\mu_1 = \mu_4 = 1$ and $\mu_2 = \mu_3 = 0$. Moreover, due to the observability singularity in $y_1 = 0$, the numerical differentiators must be switched off in the vicinity of $y_1 = 0$. In this case, the previous estimations of the derivatives are used in the set of equations (5). Moreover, due to the delay introduced by the estimation frame some extra delays of $5.10^{-4}s$ are introduced in (5) in order to compensate for the influences of the first one. Finally, a reshaping procedure is used on m_2 to overcome the distortion due to the observability singularities.
Figure 5 illustrates the chaotic behaviour of the system in the phase plot while Figures 6-9 show that one can estimate the state and recover the hidden messages. The peaks are due to the observability singularities as it is highlighted in Figure 8 where m_1, the estimate of m_1 and x_1 are given.

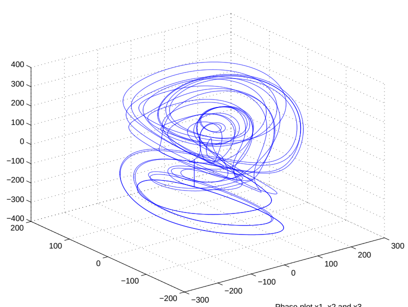

Fig. 5. Phase plot

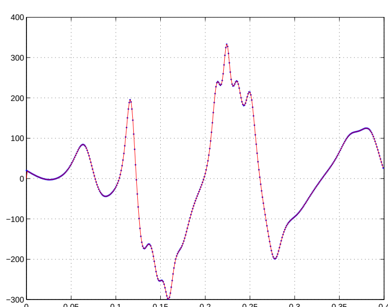

Fig. 6. x_3 in blue and its estimate in red

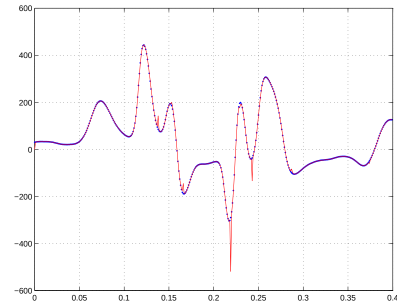

Fig. 7. x_4 in blue and its estimate in red

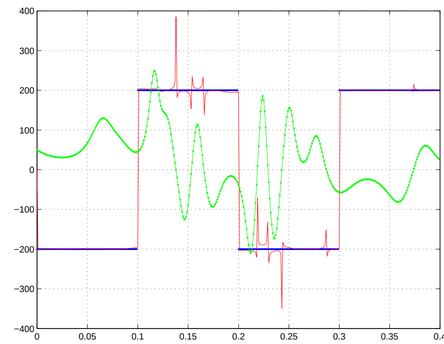

Fig. 8. m_1 in blue, its estimate in red and x_1 in green

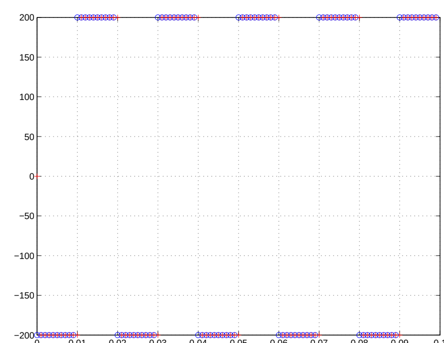

Fig. 9. m_2 in blue and its estimate in red

V. Conclusion

We considered in this paper the problem of the unknown input observer design for nonlinear systems. The corresponding conditions for observability and unknown input identifiability conditions were given in an algebraic setting. Then, it was shown that, using a new type of fast algebraic differentiators, both the state and the unknown inputs can be recovered, even when some structural conditions usually required by other methods are not fulfilled. The efficiency and robustness of the proposed scheme with respect to noise measurements were highlighted via an academic example. The practical interest of the method was also demonstrated with an application to secure communication.

References

[1] M.F. Atiyah, I.G. Macdonald, *Introduction to Commutative Algebra*, Addison-Wesley, 1969.

[2] J.-P. Barbot, T. Boukhobza, M. Djemaï, "Sliding mode observer for triangular input form", *IEEE Conf. Decision Control*, Kobe, 1996.

[3] R.W. Brockett, M.D. Mesarovic, The reproducibility of multivariable systems, *J. Math. Anal. Appl.*, vol. 11, pp. 548–563, 1965.

[4] A. Chambert-Loir, *Algèbre corporelle*, Éditions École Polytechnique, 2005. English translation: *A Field Guide to Algebra*, Springer, 2005.
[5] J. Daafouz, M. Fliess, G. Millerioux, "Une approche intrinsèque des observateurs linéaires à entrées inconnues", *Actes Conf. Internat. Automat. Francoph. - CIFA 2006*, Bordeaux, 2006 (available at `http://hal.inria.fr/inria-00001150`).
[6] E. Delaleau, P.S. Pereira da Silva, Filtrations in feedback synthesis: Part I - systems and feedbacks, *Forum Math.*, vol. 10, pp. 147–174, 1998.
[7] E. Delaleau, P.S. Pereira da Silva, Filtrations in feedback synthesis - Part II: input-output decoupling and disturbance decoupling, *Forum Math.*, vol. 10, pp. 259–276, 1998.
[8] S. Diop, M. Fliess, "On nonlinear observability", *Proc.* 1^{st} *Europ. Control Conf.*, Hermès, pp. 152–157, 1991.
[9] S. Diop, M. Fliess, "Nonlinear observability, identifiability and persistent trajectories", *Proc.* 36^{th} *IEEE Conf. Decision Control*, Brighton, pp. 714–719, 1991.
[10] D. Eisenbud, *Commutative Algebra with a View Toward Algebraic Geometry*, Springer, 1995.
[11] M. Fliess, Automatique et corps différentiels, *Forum Math.*, vol. 1, pp. 227-238, 1989.
[12] M. Fliess, Analyse non standard du bruit, *C.R. Acad. Sci. Paris Ser. 1*, vol. 342, pp. 797-802, 2006.
[13] M. Fliess, C. Join, H. Sira-Ramírez, Non-linear estimation is easy, *Int. J. Modelling Identification Control*, to appear (available at `http://hal.inria.fr/inria-00158855`).
[14] M. Fliess, J. Lévine, P. Martin, P. Rouchon, Flatness and defect of non-linear systems: introductory theory and examples, *Int. J. Control*, vol. 61, pp. 1327–1361, 1995.
[15] T. Floquet, J.-P. Barbot, W. Perruquetti, M. Djemaï, On the robust fault detection via a sliding mode disturbance observer, *Int. J. Control*, vol. 77, pp. 622-629, 2004.
[16] T. Floquet, J.P. Barbot, Super twisting algorithm based step-by-step sliding mode observers for nonlinear systems with unknown inputs, *Int. J. Systems Science*, to appear.
[17] J. P. Gauthier, H. Hammouri, S. Othman, A simple observer for nonlinear systems with applications to bioreactors, *IEEE Trans. Automat. Control*, vol. 37, pp. 875–880, 1992.
[18] H. Hammouri, M. Kinnaert, E. H. El Yaagoubi, Observer-Based Approach to Fault Detection and Isolation for Nonlinear Systems, *IEEE Trans. Automat. Control*, vol. 44, pp. 1879–1884, 1999.
[19] R. Hermann, A. J. Krener, Nonlinear controllability and observability, *IEEE Trans. Automat. Control*, vol. 22, pp. 728-740, 1977.
[20] J.E. Hershey, *Cryptography Demystified*, McGraw-Hill, 2003.
[21] M. Hou, R. J. Patton, Input observability and input reconstruction, *Automatica*, Vol. 34, pp. 789–794, 1998.
[22] M. Hou and P. C. Müller, Design of observers for linear systems with unknown inputs, *IEEE Trans. Automat. Control*, vol. 37, pp. 871–875, 1992.
[23] A. Isidori, *Nonlinear Control Systems* 3^{rd} ed., Springer, 1995.
[24] A. Isidori, C. Moog, "On the nonlinear equivalent of the notion of transmission zeros", in C. Byrnes, A. Kurzhanski (Eds), Proc. of the IIASA Conference: Modeling and Adaptive Control, pp 146–157, Springer, 1988. .
[25] E.R. Kolchin, *Differential Algebra and Algebraic Groups*, Academic Press, 1973.
[26] G. Kolumbán, M.P. Kennedy, L.O. Chua, The role of synchronization in digital communications using chaos - Part II : Chaotic modulation and chaotic synchronization, *IEEE Trans. Circuits Systems*, Part 1, vol. 44, No 10, pp. 927–936, 1997.
[27] L Kovarev, K.S. Eckert, L.O Chua, U. Parlitz, Experimental demonstration of secure communications via chaotic synchronization, *Int. J. Bifurcation Chaos*, vol. 2, pp. 709–713, 1992.
[28] A. J. Krener and W. Kang, Locally convergent nonlinear observers, *SIAM J. Control Optim.*, vol. 42, 155–177, 2003.
[29] M. Mboup, C. Join, M. Fliess, "A revised look at numerical differentiation with an application to nonlinear feedback control", *Proc.* 15^{th} *Mediterrean Conf. Control Automation – MED'2007*, Athens, 2007 (available at `http://hal.inria.fr/inria-00142588`).
[30] L. M. Pecora, T.L. Caroll, Synchronization in chaotic system, *Phys. Rev. Lett.*, vol. 64, pp. 1196–1199, 1990.
[31] W. Perruquetti, T. Floquet, P. Borne, "A note on sliding observer and controller for generalized canonical forms", *IEEE Conf. on Decision and Control*, Tampa, Florida, USA, 1998.
[32] C. de Persis, A. Isidori, A geometric approach to nonlinear fault detection and isolation, *IEEE Transactions on Automatic Control*, vol. 46, pp. 853–865, 2001.
[33] G. Qi, G. Chen, Y. Zhang, On a new asymmetric chaotic system, *Chaos Solitons Fractals*, 2007.
[34] W. Respondek, Right and left invertibility of nonlinear control systems, in *Nonlinear Controllability and Optimal Control*, H.J. Sussmann, Ed., Marcel Dekker, pp. 133–176, 1990.
[35] J. F. Ritt, *Differential algebra*, Amer. Math. Soc., 1950.
[36] J. Rudolph, *Beiträge zur flacheitsbasierten Folgeregelung linearer und nichtlinearer Syteme endlicher und undendlicher Dimension*, Shaker Verlag, 2003.
[37] M.K. Sain, J.L. Massey, Invertibility of linear time-invariant dynamical systems, *IEEE Trans. Automat. Control*, Vol. 14, pp. 141–149, 1969.
[38] L.M. Silverman, Inversion of multivariable linear systems, *IEEE Trans. Automat. Control*, vol. 14, pp. 270–276, 1969.
[39] S. N. Singh, A modified algorithm for invertibility in nonlinear systems, *IEEE Trans. Automat. Control*, vol. 26, 1981, pp. 595–598, 1981.
[40] H. Sira-Ramírez, S.K. Agrawal, *Differentially Flat Systems*, Marcel Dekker, 2004.
[41] H. Sira-Ramírez, M. Fliess, An algebraic state estimation approach for the recovery of chaotically encrypted messages, *Int. J. Bifurcation Chaos*, vol. 16, pp. 295–309, 2006.
[42] Y. Xiong, M. Saif, Sliding mode observer for nonlinear uncertain systems, *IEEE Trans. Automat. Control*, vol. 46, pp. 2012–2017, 2001.
[43] T. Yang, L. Chua, Secure communication via chaotic parameter modulation, *IEEE Trans. Circuits Systems I*, vol. 44, pp. 817–819, 1996.
[44] K. Yosida, *Operational Calculus: A Theory of Hyperfunctions* (translated from the Japanese), Springer, 1984.
[45] D.C. Youla, P. Dorato, On the inverse of linear dynamical systems, Polytechnic Institute of Brooklyn, N. Y., Electrophysics Memo pibmri-1319-66 edition, 1966 (see also P. Dorato, *IEEE Trans. Syst. Sci. Cyber.*, vol. 5, pp. 43–48, 1969).